Getting Ready for Precalculus

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Radicals

To simplify means that:

1. No radicand has a perfect square factor
2. There is no radical in the denominator

Product Property: $\sqrt{ab} = \sqrt{a}\sqrt{b}$

Quotient Property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

If the denominator contains two terms, multiply the numerator and denominator by the conjugate of the denominator (i.e. the conjugate of $3 + \sqrt{2}$ is $3 - \sqrt{2}$)

Examples:

1. Simplify $\sqrt{24}$

$$\sqrt{24} = \sqrt{4}\sqrt{6} = 2\sqrt{6}$$

2. Simplify $\sqrt{\frac{7}{2}}$

$$\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$$

Simplify each of the following.

1. $\sqrt{32}$

2. $\sqrt{(2x)^8}$

3. $\sqrt[3]{-64}$

4. $\sqrt{49m^2n^8}$

5. $\sqrt{\frac{11}{9}}$

6. $\sqrt{60}\sqrt{105}$

7. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

8. $\frac{1}{\sqrt{2}}$

9. $\frac{2}{\sqrt{3}}$

10. $\frac{3}{2 - \sqrt{5}}$

Geometry

Pythagorean Theorem: $a^2 + b^2 = c^2$

Special Triangles:

Triangle with $45°$ angles, legs labeled a and a, hypotenuse $a\sqrt{2}$.

Triangle with $60°$ and $30°$ angles, short leg b, long leg $b\sqrt{3}$, hypotenuse $2b$.
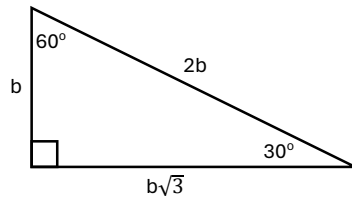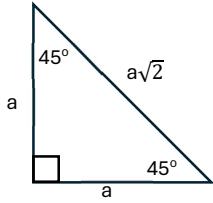
Find the value of the missing variables.

1.

Right triangle with vertical leg 9, horizontal leg 12, hypotenuse a.

2.

Square with sides 5 and 5, diagonal b.

3.

Isosceles triangle with height 8, base 12, slant side x.

4.

Right triangle with $45°$ angle, vertical leg w, hypotenuse y, horizontal leg $\sqrt{2}$.

5.

Right triangle with $45°$ angle, side 4, vertical leg p, horizontal leg n.

6.

Right triangle with $60°$ angle, hypotenuse 4, vertical leg m, horizontal leg k.

7.

60°

√3

h

g

8.

9

a

12
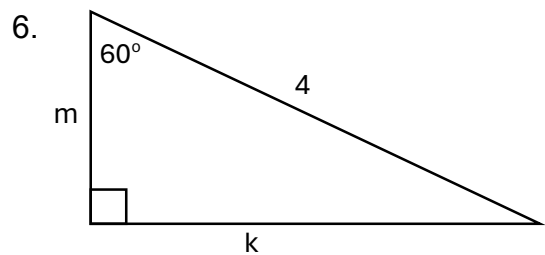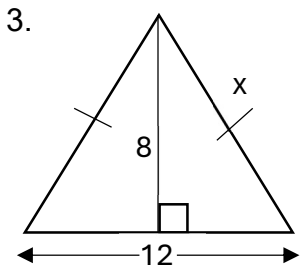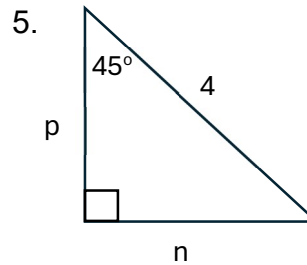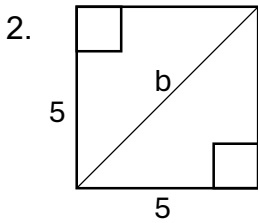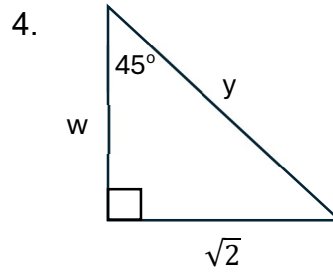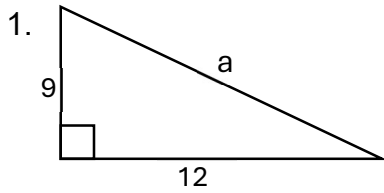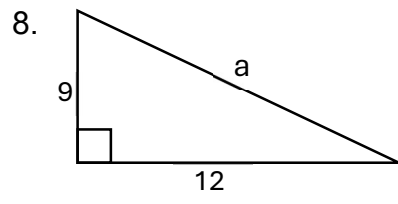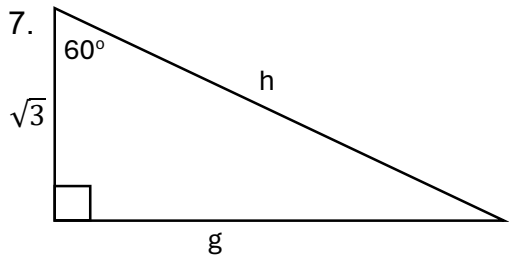
Writing Linear Equations in Slope-Intercept Form

Slope-Intercept Form: $y = mx + b$

Point-Slope Form: $y - y_1 = m(x - x_1)$

Standard Form: $Ax + By = C$

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Write an equation in slope-intercept form with the following conditions.

1. Through (-3, -8) and a slope of -5

2. Through (4, 3) and (7, -2)

3. x-intercept = 3 and y-intercept = 2

Write an equation in point-slope form with the following conditions.

4. Through (-3, -8) and a slope of -5

5. Through (0, 2); perpendicular to $y = -4x + 6$

6. Through (2, -3); parallel to $y = -2x - 3$

Complete the following.

7. State the slope and y-intercept of:
$5x - 4y = 8$

8. Find the x-intercept and y-intercept of the equation: $2x - y = 5$

9. Write the following equation in Standard Form: $y = 7x - 5$

10. Write the following equation in Slope-Intercept Form: $3x + 4y = 6$

11. Write the following equation in Point-Slope Form: $2x - 5y = 7$

Solving Linear Systems

Solve the linear system by substitution:

$$3x + y = 6$$

$$2x - 2y = 4$$

$y = -3x + 6$	**Solve 1ˢᵗ equation for y**
$2x - 2(-3x + 6) = 4$	**Substitute into 2ⁿᵈ equation**
$x = 2$	**Solve for x**
$y = 0$	**Substitute back into either equation**

Solution: (2, 0)

Solve the linear system by elimination:

$$2x + 3y = 13$$

$$-5x + 4y = -17$$

$\begin{array}{c} 4x + 6y = 26 \\ -4x + 3y = 1 \\ \hline 9y = 27 \end{array}$	**Multiply 1ˢᵗ equation by 2 and add**
$y = 3$	**Solve for y**
$2x + 3(3) = 13$	**Substitute y into the 1ˢᵗ equation**
$x = 2$	**Solve for x**

Solution: (2, 3)

Solve the linear system using substitution.

1. $y = 2x - 7$
   $x + 2y = 1$

3. $2x + y = -15$
   $y - 5x = 6$

2. $x + 4y = 9$
   $x - y = 4$

4. $2x + y = 4$
   $3x - y = 14$

Solve the linear system using elimination.

5.
$$-x + y = -4$$
$$2x - 3y = 5$$

7.
$$3x - 5y = -7$$
$$-4x + 7y = 8$$

6.
$$x + 6y = 28$$
$$2x - 3y = -19$$

8.
$$8x - 7y = -3$$
$$6x - 5y = -1$$

Applying Exponent Properties

Exponent Rules:

Product: $x^a x^b = x^{a+b}$

Quotient: $\dfrac{x^c}{x^d} = x^{c-d}$

Power: $(x^e)^f = x^{ef}$

Zero: $x^0 = 1$

Negative: $x^{-g} = \dfrac{1}{x^g}$ and $\dfrac{1}{x^{-g}} = x^g$

Ex 1 - $\left(\dfrac{x^3}{y}\right)^4 \cdot \dfrac{2}{x^5}$

$\dfrac{(x^3)^4}{y^4} \cdot \dfrac{2}{x^5}$	**Power property**
$\dfrac{x^{12}}{y^4} \cdot \dfrac{2}{x^5}$	**Power property**
$\dfrac{2x^{12}}{x^5 y^4}$	**Multiply fractions**
$\dfrac{2x^7}{y^4}$	**Quotient Property**

Simplify the expression.

1. $4^3 \cdot 4^3$

2. $z^3 \cdot z^5 \cdot z^5$

3. $(y^4)^5$

4. $-(8xy)^2$

5. $(2x^2)^4 \cdot x^5$

6. $\dfrac{(-3)^7}{(-3)^4}$

7. $\dfrac{5^2 \cdot 5^4}{5^3}$

8. $\dfrac{6}{7r^{10}} \cdot \left(\dfrac{r^5}{p}\right)^5$

9. $\left(\frac{7x^5y^0}{y^2}\right)^3$

11. 3^{-2}

10. $7^{-5} \cdot 7^5$

12. $\frac{x^{-2}}{xy^2}$

Combining Polynomials

Adding/Subtracting polynomials

Ex 1:

$(2x^3 + 4x^2 + 1) + (5x^2 + x + 4) = 2x^3 + 9x^2 + x + 5$

Ex 2:

$(3x^2 + 2) - (4x^2 - x - 9) = -x^2 + x + 11$

Multiplying polynomials

$(2x^2 + 4x + 1)(3x - 2)$	**Original product**
$6x^3 - 4x^2 + 12x^2 - 8x + 3x - 2$	**Distributive Property**
$6x^3 + 8x^2 - 5x - 2$	**Combine like terms**

Keep terms in descending order of degree

Find the sum or difference.

1. $(9x + 6x^3 - 8x^2) + (-5x^3 + 6x)$

2. $(7a^3 - 4a^2 - 2a + 1) + (a^3 - 1)$

3. $(11y^5 + 3x^2 - 4) + (y^2 - y + 1)$

4. $(3n^2 - 4n + 1) - (8n^2 - 4n + 17)$

5. $(2b^3 + 8) - (-3b^3 + 7b - 5)$

6. $(-k^2 + 7k + 5) - (2k^4 - 3k^3 - 6)$

Find the product in simplest terms.

7. $(3y + 4)(y + 2)$

10. $(x^2 + 3x - 1)(x + 7)$

8. $(2x^2 + x)(x - 3)$

11. $(a^2 + 4)(2a^2 - 2a - 4)$

9. $(5b - 1)(b^2 + 6)$

12. $(w^2 - 4w + 2)(3w^2 + 2w - 5)$

Factoring Polynomials and Solving Polynomial Equations

Special Factoring Rules:

$$x^2 - y^2 = (x - y)(x + y)$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Factor the following polynomials.

Ex 1 $- y^2 + 15y + 26$

$y^2 + 15y + 26$	**Write original problem**
$(y + 13)(y + 2)$	**Factor**

Ex 2 $- x^3 + 3x^2 + 9x + 27$

$(x^3 + 3x^2) + (9x + 27)$	**Pair first two and last two terms**
$x^2(x + 3) + 9(x + 3)$	**Factor the GCF from each pair**
$(x + 3)(x^2 + 9)$	**Factor out common term**

Solve the following equation.

Ex 3 $- 6x^2 + 42x = 0$

$6x^2 + 42x = 0$	**Write original equation**
$6x(x + 7) = 0$	**Factor left side**
$6x = 0$ or $x + 7 = 0$	**Zero Product Property**
$x = 0$ or $x = -7$	**Solve for x**

Factor the polynomials.

1. $x^2 + 4x - 12$

2. $6 - 5x - x^2$

3. $9n^2 - 4$

4. $2k^2 + 2n - 60$

5. $-10b^4 - 15b^2$

6. $9a^2 + 30a + 25$

7. $27w^3 - 8$

13

Solve the following polynomial equations.

8. $2a^2 + 26a = 0$

12. $x^2 + 25 = 10x$

9. $5y^2 = -50y$

13. $n^2 - 14n + 40 = 0$

10. $w^2 - 4 = 0$

14. $2x^3 + 5x^2 + 6x + 15 = 0$

11. $b^2 - 4b - 12 = 0$

Multiplying and Dividing Rational Expressions

Factor the numerator and denominator completely and cancel any common factors.

When dividing, multiply by the reciprocal of the second expression.

Simplify.

Ex 1 $- \dfrac{y^2+10y+}{5-4y-y^2} \div \dfrac{y^3+4y^2-21y}{y^2+2y-15}$

$\dfrac{y^2+10y+21}{5-4y-y^2} \div \dfrac{y^3+4y^2-21y}{y^2+2y-1}$	**Write original problem**
$\dfrac{y^2+10y+21}{5-4y-y^2} \cdot \dfrac{y^2+2y-15}{y^3+4y^2-21y}$	**Multiply by the reciprocal**
$\dfrac{(y+7)(y+3)}{(y+5)(-y+1)} \cdot \dfrac{(y+5)(y-3)}{y(y-3)(y+7)}$	**Factor completely**
$\dfrac{(y+3)}{y(-y+1)}$	**Cancel out common factors**

Simplify the rational expressions.

1. $\dfrac{5z^3+z^2-z}{3z}$

2. $\dfrac{m^2-25}{m^2+5m}$

3. $\dfrac{10r^5}{21\ ^2} \cdot \dfrac{3q}{5r^3}$

4. $\dfrac{a^2-5a+6}{a+4} \cdot \dfrac{3a+12}{a-2}$

5. $\dfrac{3n^2-27n}{10n} \div \dfrac{24n+2}{15}$

7. $\dfrac{3x^2-27}{x^2-12x+35} \cdot \dfrac{x^2-x-20}{x+3}$

6. $\dfrac{8k^3-16k^2}{k-5} \div \dfrac{40\ ^3+16k^2}{5k+2}$

8. $\dfrac{x^2-3x-2}{x-5} \cdot \dfrac{5x-25}{x^2-49}$

Simplifying and Solving Rational Expressions

When adding or subtracting rational expressions,

- Factor completely
- Find the common denominator
- Add/subtract the numerators
- Simplify

When solving rational expressions, multiply each term by the least common denominator of all the fractions and solve. Make sure to check for extraneous solutions.

Simplify.

Ex 1 $- \frac{3x+1}{x^2+2x} + \frac{5x-4}{2x+4}$

$\frac{3x+1}{x^2+2x} + \frac{5x-4}{2x+4}$ **Write original problem**

$\frac{3x+1}{x(x+2)} + \frac{5x-4}{2(x+2)}$ **Factor completly**

$\frac{2(3x+1)}{2x(x+2)} \cdot \frac{x(5x-4)}{2x(x+2)}$ **Rewrite each fraction with the LCD**

$\frac{6x+2+5x^2-4x}{2x(x+2)}$ **Write as one fraction**

$\frac{5x^2+2x+2}{2x(x+2)}$ **Simplify**

Simplify the rational expressions.

1. $\frac{2x}{5} - \frac{x}{3}$

3. $\frac{2-a^2}{a^2+a} + \frac{3a+4}{3a+3}$

2. $\frac{b-a}{a^2 b} + \frac{a+b}{ab^2}$

4. $\frac{3x-8}{x^2+6x+8} + \frac{2x-3}{x^2+3x+2}$

Solve each equation. Check for extraneous solutions.

5. $\dfrac{12}{x} + \dfrac{3}{4} = \dfrac{3}{2}$

7. $\dfrac{1}{x-2} + \dfrac{1}{x^2-7x+1} = \dfrac{6}{x-2}$

6. $\dfrac{x+10}{x^2-2} = \dfrac{4}{x}$

8. $\dfrac{5}{x^3+5x^2} = \dfrac{4}{x+5} + \dfrac{1}{x^2}$