Getting Ready for AP Calculus AB

1. Factoring Polynomials and Solving Polynomial Equations
2. Function Families and Characteristics
3. Polynomial Long Division
4. Logarithmic Properties
5. Unit Circle and Trigonometric Evaluation
6. Difference Quotient
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Factoring Polynomials and Solving Polynomial Equations

Special Factoring Rules:

$$x^2 - y^2 = (x - y)(x + y)$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Factor the following polynomials.

Ex 1 – $y^2 + 15y + 26$

$y^2 + 15y + 26$	**Write original problem**
$(y + 13)(y + 2)$	**Factor**

Ex 2 – $x^3 + 3x^2 + 9x + 27$

$(x^3 + 3x^2) + (9x + 27)$	**Pair first two and last two terms**
$x^2(x + 3) + 9(x + 3)$	**Factor the GCF from each pair**
$(x + 3)(x^2 + 9)$	**Factor out common term**

Solve the following equation.

Ex 3 – $6x^2 + 42x = 0$

$6x^2 + 42x = 0$	**Write original equation**
$6x(x + 7) = 0$	**Factor left side**
$6x = 0$ or $x + 7 = 0$	**Zero Product Property**
$x = 0$ or $x = -7$	**Solve for x**

Factor the polynomials.

1. $x^2 + 4x - 12$

2. $6 - 5x - x^2$

3. $9n^2 - 4$

4. $2k^2 + 2n - 60$

5. $-10b^4 - 15b^2$

6. $9a^2 + 30a + 25$

7. $27w^3 - 8$

Solve the following polynomial equations.

8. $2a^2 + 26a = 0$

9. $5y^2 = -50y$

10. $w^2 - 4 = 0$

11. $b^2 - 4b - 12 = 0$

12. $x^2 + 25 = 10x$

13. $n^2 - 14n + 40 = 0$

14. $2x^3 + 5x^2 + 6x + 15 = 0$

Function Families and Characteristics

Polynomial Functions:

- $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$
- x-intercepts: largest number of possible zeros is equal to its degree
- y-intercepts: x = 0

Exponential Functions:

- $f(x) = ab^x$
- No x-intercepts
- y-intercept: occurs at the coordinate (o, a)

Trigonometric Functions:

- $f(x) = \sin x$
- $f(x) = \cos x$
- $f(x) = \tan x$

Ex - Find the x-intercept(s) and y-intercepts of

$$f(x) = 4x^3 + 12x^2 - 7x - 21$$

x-intercepts:

$4x^3 + 12x^2 - 7x - 21 = 0$	**Set $f(x) = 0$**
$4x^2(x + 3) - 7(x + 3) = 0$	**Factor by grouping**
$(x + 3)(4x^2 - 7) = 0$	**Factor**
$x = -3$, $x = -\frac{7}{4}$, and $x = \frac{7}{4}$	**Zero Product Principle**

y-intercept:

$$f(0) = -21$$

Factor and find the x-intercept(s) with one given intercept.

1. $f(x) = 5x^3 + 4x^2 - 20x - 16\,;\, 2$
2. $g(x) = 2x^4 - x^3 - 18x^2 + 9x\,;\, -3$
3. $h(x) = 3x^3 + 4x^2 - 35x - 12\,;\, 3$
4. $j(x) = 3x^5 - 9x^4 - 12x^3\,;\, -1$

Determine the domain and range for each function.

1. $f(x) = x^2 - 4$

2. $g(x) = \frac{1}{2}e^x$

3. $h(x) = \sin\left(\frac{\pi}{2}x\right)$

4. $j(x) = 2x^4 - 3x^3 + x^2 - 9x + 1$

5. $f(x) = \tan\left(\frac{x}{3}\right)$

6. $k(x) = \frac{1}{(x-4)^2} + x$

Polynomial Long Division

Dividing polynomials creates a new function. The last term of the new function is the remainder of the divisor.

Ex - $(x^4 + 5x^3 - 11x^2 - 25x + 29) \div (x + 6)$

$$= x^3 - x^2 - 5x + 5 - \frac{1}{x + 6}$$

Divide; show your answers with a remainder over its divisor.

1. $(x^4 + 11x^3 + 33x^2 + 24x + 32) \div (x + 6)$

2. $(4t^3 - 9t^2 + 9t + 3) \div (t - 1)$

3. $(6x^3 + 20x^2 - 15x + 9) \div (x + 4)$

4. $(8x^5 + 32x^4 + 5x + 20) \div (x + 4)$

5. $(t^5 + 8t^4 - 13t^3 - 5t^2 - 8t + 14) \div (t + 8)$

6. $(p^3 - 10p^2 + 20p + 26) \div (p - 5)$

Exponential and Logarithmic Properties

Logarithmic Rules:	**Exponential Rules:**
$\log_a(xy) = \log_a x + \log_a y$	$x^a x^b = x^{a+b}$
$\log_a \frac{x}{y} = \log_a x - \log_a y$	$\frac{x^a}{x^b} = x^{a-b}$
$\log_a(x)^b = b \log_a x$	$(x^a y^b)^c = x^{ac} y^{bc}$

Expand each logarithm.

1. $\log(3x)^4$

2. $\log \frac{x}{y^5}$

3. $\ln(xyz^2)$

4. $\log \frac{x^4}{y^3}$

5. $\ln(5e)^5$

6. $\log \sqrt[3]{xy^2z^5}$

Condense each expression to a single logarithm.

7. $\log 2 + \log 11 + \log 7$

8. $\ln x - 5 \ln y$

9. $4\log x + 5 \log y - 6 \log z$

10. $\frac{2}{3}\log 8 + \frac{4}{7}\log t$

Simplify each exponential expression.

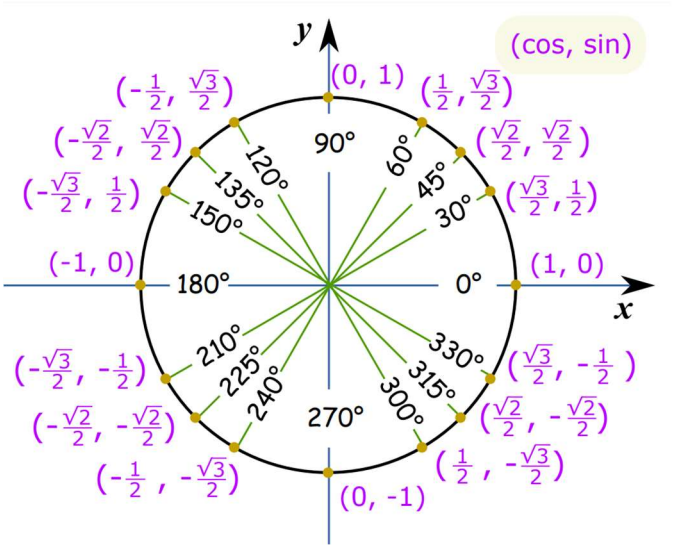
11. $(w^4 w^8)^{-2}$

12. $x^{-3}y^2z^{-1}x^7y^{-8}z$

13. $3y^2z^{-12} \cdot \frac{x^2y^4}{5z^{-7}}$

14. $(a^2b^3c^4)^5(a^{-6}b^{-7}c^{-8})^{-9}$

Unit Circle



Reciprocal Functions:

$$cscx = \frac{1}{sinx}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Evaluate the following trigonometric expressions.

1. $\sin \pi$.
2. $\cos \frac{\pi}{3}$
3. $\tan \frac{\pi}{6}$
4. $\csc \frac{3\pi}{4}$
5. $\cot \frac{-7\pi}{4}$
6. $\sec \frac{\pi}{4}$
7. $\sin \frac{-\pi}{2}$
8. $\cos \frac{13\pi}{4}$

Difference Quotient

Difference Quotient:

$$\frac{f(x+h)-f(x)}{h}$$

Ex - $f(x)=3x^2-5x+4$

$$\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)^2-5(x+h)+4-(3x^2-5x+4)}{h}=\frac{3x^2+6xh+3h^2-5x-5h+4-3x^2+5x-4}{h}=\frac{6xh+3h^2-5h}{h}=6x+3h-5$$

Find the difference quotient for each function.

1. $f(x)=2x^3-4x^2+5x$

2. $g(x)=-4x$

3. $j(x) = x^2 + 8x - 11$

4. $f(x) = \frac{1}{x}$

5. $g(x) = \frac{1}{x^2}$

Using the TI-84 Calculator

Finding zeros, maximums, and minimums

1. Enter function into Y = menu
2. 2nd TRACE → 2: zero, 3: minimum, or 4: maximum
3. Left bound: move cursor to the left of the zero, minimum, or maximum, then press ENTER
4. Right bound: move cursor to the right of the zero, minimum, or maximum, then press ENTER
5. Guess: Press ENTER

Finding points of intersection

1. Enter function one into Y1 and function two into Y2
2. 2nd TRACE → 5: intersect
3. Move cursor close to the point of intersection
4. Press ENTER on the first curve
5. Press ENTER on the second curve
6. Guess by pressing ENTER

Using a graphing calculator, find the maximums, minimums, and zeros for each function over the given interval. Round your answers to three decimal places.

1. $f(x) = 4x^3 + 8x^2 - 2\,;[-2,2]$

2. $g(x) = \frac{1}{2}x^4 - 2x^3 + 3x - 1\,;[-3,5]$

3. $j(x) = \frac{3}{2}e^x - x^2 + \left(\frac{1}{4}\right)^x - 6\,;[-2,2]$

4. $h(x) = \tan(\sin x^2)\,;[-1,3]$

Using a graphing calculator, find the points of intersection of the two given functions. Round your answers to three decimal places.

5. $f(x) = \sin x \ ; g(x) = e^x - 4$

6. $f(x) = \ln \frac{1}{x+2} \ ; g(x) = x^3 - 4x^2 + 3$

7. $f(x) = xe^{\frac{-x}{5}} \ ; g(x) = x^2$

8. $f(x) = x \cos\left(x^{\frac{2}{3}}\right) + x^{\frac{2}{3}} \sin(x^2) \ ; g(x) = x^3$